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SADIK TRANSFORM OF ERROR FUNCTION (PROBABILITY INTEGRAL)

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ABSTRACT

The solutions of many advanced engineering problems like Fick's second law, heat and mass transfer problems, vibrating beams problems contains error and complementary error function. When we use any integral transform to solve these types of problems, it is very necessary to know the integral transform of error function. In this article, we find the Sadik transform of error and complementary error functions. To demonstrate the usefulness of Sadik transform of error function, some numerical applications are considered in application section for solving improper integrals which contain error function. It is pointed out that Sadik transform give the exact solution of improper integral which contains error function without any tedious calculation work.

Keywords: *Sadik transform, Error function, Complementary error function, Improper integral.*

AMS SUBJECT CLASSIFICATION: 44A05, 44A20, 44A35

I. INTRODUCTION

Integral transforms are highly efficient for solving many advance problems of science and engineering such as radioactive decay problems, heat conduction problems, problem of motion of a particle under gravity, vibration problems of beam, electric circuit problems and population growth problems. Many researchers applied different integral transforms (Laplace transform [1-2], Fourier transform [2], Kamal transform [3-10, 48-49], Aboodh transform [11-16, 50-53], Mahgoub transform [17-25, 45-47], Mohand transform [26-29, 36, 54-56], Elzaki transform [37-40, 57-59], Shehu transform [41-43, 60] Sumudu transform [44, 61-62] and Sadik transform [63-65]) and solved differential equations, delay differential equations, partial differential equations, integral equations, integro-differential equations and partial integro-differential equations. Sudhanshu et al. [30-35] discussed the comparative study of Mohand and other transforms (Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform and Mahgoub transform).

Error function occurs frequently in probability, physics, thermodynamics, statistics, mathematics and many engineering problems like heat conduction problems, vibrating beams problems etc. The error function is also known as the probability integral. The error function is a special function because it cannot be evaluated by usual methods of integration. Mathematically error and complimentary error functions are defined by [66-71].

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \quad (1)$$

and

$$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt \quad (2)$$

The Sadik transform of the function $F(t)$ for all $t \geq 0$ is defined as [63]:

$$S\{F(t)\} = \frac{1}{v^\beta}\int_0^\infty F(t)e^{-tv^\alpha}dt = T(v^\alpha, \beta), \quad (3)$$

where v is complex variable and $\alpha \neq 0$ & β are any real numbers. Here S is called the Sadik transform operator.

The main purpose of the present article is to determine Sadik transform of error function and explain the importance of Sadik transform of error function by giving some numerical applications in application section of this paper.

II. SOME USEFUL PROPERTIES OF SADIK TRANSFORM

2.1 Linearity property of Sadik transforms:

If Sadik transform of functions $F_1(t)$ and $F_2(t)$ are $T_1(v^\alpha,\beta)$and $T_2(v^\alpha,\beta)$ respectively then Shehu transform of $[aF_1(t)+bF_2(t)]$ is given by $[a\,T_1(v^\alpha,\beta)+bT_2(v^\alpha,\beta)]$, where a,b are arbitrary constants.

Proof: By the definition of Sadik transform, we have

$$S\{F(t)\}=\frac{1}{v^\beta}\int_0^\infty F(t)e^{-tv^\alpha}dt$$

$$\Rightarrow S\{aF_1(t)+bF_2(t)\}=\frac{1}{v^\beta}\int_0^\infty [aF_1(t)+bF_2(t)]e^{-tv^\alpha}dt$$

$$\Rightarrow S\{aF_1(t)+bF_2(t)\}=a\left[\frac{1}{v^\beta}\int_0^\infty F_1(t)e^{-tv^\alpha}dt\right]+b\left[\frac{1}{v^\beta}\int_0^\infty F_2(t)e^{-tv^\alpha}dt\right]$$

$$\Rightarrow S\{aF_1(t)+bF_2(t)\}=aS\{F_1(t)\}+bS\{F_2(t)\}$$

$\Rightarrow S\{aF_1(t)+bF_2(t)\}=a\,T_1(v^\alpha,\beta)+bT_2(v^\alpha,\beta)$, where a,b are arbitrary constants.

2.2 Change of scale property of Sadik transforms:

If Sadik transform of function $F(t)$ is $T(v^\alpha,\beta)$ then Sadik transform of function $F(at)$ is given by $\frac{1}{a}T\left(\frac{v^\alpha}{a},\beta\right)$.

Proof: By the definition of Sadik transform, we have

$$S\{F(at)\}=\frac{1}{v^\beta}\int_0^\infty F(at)e^{-tv^\alpha}dt \quad (4)$$

Put $at=p \Rightarrow adt=dp$ in equation (4), we have

$$S\{F(at)\}=\frac{1}{a}.\frac{1}{v^\beta}\int_0^\infty F(p)e^{-\frac{pv^\alpha}{a}}dp$$

$$\Rightarrow S\{F(at)\}=\frac{1}{a}\left[\frac{1}{v^\beta}\int_0^\infty F(p)e^{-p\left(\frac{v^\alpha}{a}\right)}dp\right]$$

$$\Rightarrow S\{F(at)\}=\frac{1}{a}T\left(\frac{v^\alpha}{a},\beta\right)$$

2.3 Shifting property of Sadik transform:

If Sadik transform of function $F(t)$ is $T(v^\alpha,\beta)$ then Sadik transform of function $e^{at}F(t)$is given by $T(v^\alpha-a,\beta)$.

Proof: By the definition of Sadik transform, we have

$$S\{e^{at}F(t)\}=\frac{1}{v^\beta}\int_0^\infty e^{at}F(at)e^{-tv^\alpha}dt$$

$$\Rightarrow S\{e^{at}F(t)\}=\frac{1}{v^\beta}\int_0^\infty F(t)e^{-(v^\alpha-a)t}dt$$

$$\Rightarrow S\{e^{at}F(t)\}=T(v^\alpha-a,\beta)$$

2.4 Sadik transform of the derivatives of the function $F(t)$ [63-64]:

If $S\{F(t)\}=T(v^\alpha,\beta)$ then

a) $S\{F'(t)\}=v^\alpha T(v^\alpha,\beta)-\frac{F(0)}{v^\beta}$

b) $S\{F''(t)\} = v^{2\alpha}T(v^\alpha, \beta) - \frac{F'(0)}{v^\beta} - v^\alpha \frac{F(0)}{v^\beta}$

c) $S\{F^{(n)}(t)\} = v^{n\alpha}T(v^\alpha, \beta) - v^{(n-1)\alpha}\frac{F(0)}{v^\beta} - v^{(n-2)\alpha}\frac{F'(0)}{v^\beta} - \cdots \ldots - \frac{F^{(n-1)}(0)}{v^\beta}$.

2.5 Sadik transform of integral of a function $F(t)$:

If $S\{F(t)\} = T(v^\alpha, \beta)$ then

$S\left\{\int_0^t F(t)dt\right\} = \frac{1}{v^\alpha}T(v^\alpha, \beta)$.

Proof: Let $G(t) = \int_0^t F(t)dt$. Then $G'(t) = F(t)$and $G(0) = 0$.

Now by the property of Sadik transform of the derivative of function, we have

$$S\{G'(t)\} = v^\alpha S\{G(t)\} - \frac{G(0)}{v^\beta} = v^\alpha S\{G(t)\}$$

$$\Rightarrow S\{G(t)\} = \frac{1}{v^\alpha}S\{G'(t)\} = \frac{1}{v^\alpha}\ S\{F(t)\}$$

$$\Rightarrow S\{G(t)\} = \frac{1}{v^\alpha}T(v^\alpha, \beta)$$

$$\Rightarrow S\left\{\int_0^t F(t)dt\right\} = \frac{1}{v^\alpha}T(v^\alpha, \beta)$$

2.6 Convolution theorem for Sadik transforms:

If Sadik transform of functions $F_1(t)$ and $F_2(t)$are $T_1(v^\alpha, \beta)$and $T_2(v^\alpha, \beta)$ respectively then Sadik transform of their convolution $F_1(t) * F_2(t)$ is given by

$S\{F_1(t) * F_2(t)\} = v^\beta S\{F_1(t)\}S\{F_2(t)\}$

$\Rightarrow S\{F_1(t) * F_2(t)\} = v^\beta\ T_1(v^\alpha, \beta)T_2(v^\alpha, \beta)$, where $F_1(t) * F_2(t)$ is defined by

$F_1(t) * F_2(t) = \int_0^t F_1(t-x)\, F_2(x)dx = \int_0^t F_1(x)\, F_2(t-x)dx$.

Proof: By the definition of Sadik transform, we have

$$S\{F_1(t) * F_2(t)\} = \frac{1}{v^\beta}\int_0^\infty [F_1(t) * F_2(t)]e^{-tv^\alpha}dt$$

$$\Rightarrow S\{F_1(t) * F_2(t)\} = \frac{1}{v^\beta}\int_0^\infty e^{-tv^\alpha}\left[\int_0^t F_1(t-x)\, F_2(x)dx\right]dt$$

By changing the order of integration, we have

$$S\{F_1(t) * F_2(t)\} = \int_0^\infty F_2(x)\left[\frac{1}{v^\beta}\int_x^\infty e^{-tv^\alpha}F_1(t-x)\, dt\right]dx$$

Put $t - x = p$ so that $dt = dp$ in above equation, we have

$$S\{F_1(t) * F_2(t)\} = \int_0^\infty F_2(x)\left[\frac{1}{v^\beta}\int_0^\infty e^{-(p+x)v^\alpha}F_1(p)\, dp\right]dx$$

$$\Rightarrow S\{F_1(t) * F_2(t)\} = \int_0^\infty F_2(x)e^{-xv^\alpha}\left[\frac{1}{v^\beta}\int_0^\infty e^{-pv^\alpha}F_1(p)\, dp\right]dx$$

$$= \int_0^\infty F_2(x)e^{-xv^\alpha}\,[S\{F_1(t)\}]dx = [S\{F_1(t)\}]\int_0^\infty F_2(x)e^{-xv^\alpha}\, dx$$

$$= [\,T_1(v^\alpha, \beta)]v^\beta\left[\frac{1}{v^\beta}\int_0^\infty F_2(x)e^{-xv^\alpha}\, dx\right] = v^\beta S\{F_1(t)\}S\{F_2(t)\}$$

$$\Rightarrow S\{F_1(t) * F_2(t)\} = v^\beta\ T_1(v^\alpha, \beta)T_2(v^\alpha, \beta).$$

III. SADIK TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [63, 65]

Table: 1

S.N.	$F(t)$	$S\{F(t)\} = T(v^\alpha, \beta)$

1.	1	$\frac{1}{v^{\alpha+\beta}}$
2.	t	$\frac{1}{v^{2\alpha+\beta}}$
3.	t^2	$\frac{2!}{v^{3\alpha+\beta}}$
4.	$t^n, n \in N$	$\frac{n!}{v^{(n+1)\alpha+\beta}}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{(n+1)\alpha+\beta}}$
6.	e^{at}	$\frac{1}{v^{\beta}(v^{\alpha}-a)}$
7.	$sinat$	$\frac{a}{v^{\beta}(v^{2\alpha}+a^2)}$
8.	$cosat$	$\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+a^2)}$
9.	$sinhat$	$\frac{a}{v^{\beta}(v^{2\alpha}-a^2)}$
10.	$coshat$	$\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}-a^2)}$

IV. SOME IMPORTANT PROPERTIES OF ERROR AND COMPLEMENTARY ERROR FUNCTIONS

4.1 The sum of error and complementary error functions is unity:

$erf(x) + erfc(x) = 1$

Proof: we have $\int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$

$$\Rightarrow \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-t^2} dt = 1$$

$$\Rightarrow \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt + \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt = 1$$

$\Rightarrow erf(x) + erfc(x) = 1$

4.2 Error function is an odd function:

$erf(-x) = -erf(x)$

4.3 The value of error function at $x = 0$ is 0:

$erf(0) = 0.$

4.4 The value of complementary error function at $x = 0$ is 1:

$erfc(0) = 1.$

4.5 The domain of error and complementary error functions is $(-\infty, \infty)$.

4.6 $\boldsymbol{erf(x) \rightarrow 1 as\ x \rightarrow \infty}$.

4.7 $erfc(x) \to 0$ as $x \to \infty$.

4.8 The value of error functions $erf(x)$ for different values of x [67]:

Table: 2

S.N.	x	$erf(x)$
1.	0.00	0.00000
2.	0.02	0.02256
3.	0.04	0.04511
4.	0.06	0.06762
5.	0.08	0.09008
6.	0.10	0.11246
7.	0.12	0.13476
8.	0.14	0.15695
9.	0.16	0.17901
10.	0.18	0.20094
11.	0.20	0.22270

V. SADIK TRANSFORM OF ERROR FUNCTION

By equation (1), we have

$$erf(\sqrt{t}) = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-x^2}dx$$

$$= \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}}\left[1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots \ldots\right]dx$$

$$= \frac{2}{\sqrt{\pi}}\left[x - \frac{x^3}{3.1!} + \frac{x^5}{5.2!} - \frac{x^7}{7.3!} + \cdots \ldots\right]_0^{\sqrt{t}}$$

$$= \frac{2}{\sqrt{\pi}}\left[t^{1/2} - \frac{t^{3/2}}{3.1!} + \frac{t^{5/2}}{5.2!} - \frac{t^{7/2}}{7.3!} + ..\right] \quad (5)$$

Applying Sadik transform both sides on equation (5), we get

$$S\{erf(\sqrt{t})\} = \frac{2}{\sqrt{\pi}}S\left\{\left[t^{1/2} - \frac{t^{3/2}}{3.1!} + \frac{t^{5/2}}{5.2!} - \frac{t^{7/2}}{7.3!} + ..\right]\right\} \quad (6)$$

Applying the linearity property of Sadik transform on equation (6), we get

$$S\{erf(\sqrt{t})\} = \frac{2}{\sqrt{\pi}}\left[\frac{\Gamma\left(\frac{3}{2}\right)}{v^{(3/2)\alpha+\beta}} - \frac{\Gamma\left(\frac{5}{2}\right)}{3.1!}\left[\frac{1}{v^{(5/2)\alpha+\beta}}\right] + \frac{\Gamma\left(\frac{7}{2}\right)}{5.2!}\left[\frac{1}{v^{(7/2)\alpha+\beta}}\right] - \frac{\Gamma\left(\frac{9}{2}\right)}{7.3!}\left[\frac{1}{v^{(9/2)\alpha+\beta}}\right] + \cdots\right]$$

$$= \frac{2}{\sqrt{\pi}}\frac{\Gamma\left(\frac{3}{2}\right)}{v^{(3/2)\alpha+\beta}}\left[1 - \frac{1}{2}\left(\frac{1}{v^{\alpha}}\right) + \frac{1.3}{2.4}\left(\frac{1}{v^{\alpha}}\right)^2 - \frac{1.3.5}{2.4.6}\left(\frac{1}{v^{\alpha}}\right)^3 + \cdots \ldots \ldots\right]$$

$$= \frac{1}{v^{(3/2)\alpha+\beta}}\left(1 + \frac{1}{v^{\alpha}}\right)^{-1/2} = \frac{1}{v^{\alpha+\beta}\sqrt{(1+v^{\alpha})}} \quad (7)$$

VI. SADIK TRANSFORM OF COMPLEMENTARY ERROR FUNCTION

We have, $erf(\sqrt{t}) + erfc(\sqrt{t}) = 1$

$$\Rightarrow erfc(\sqrt{t}) = 1 - erf(\sqrt{t}) \quad (8)$$

Applying Sadik transform both sides on equation (8), we have

$$S\{erfc(\sqrt{t})\} = S\{1 - erf(\sqrt{t})\} \tag{9}$$

Applying the linearity property of Sadik transform on equation (9), we get

$$S\{erfc(\sqrt{t})\} = S\{1\} - S\{erf(\sqrt{t})\}$$

$$\Rightarrow S\{erfc(\sqrt{t})\} = \frac{1}{v^{\alpha+\beta}} - \frac{1}{v^{\alpha+\beta}\sqrt{(1+v^{\alpha})}}$$

$$\Rightarrow S\{erfc(\sqrt{t})\} = \frac{1}{v^{\alpha+\beta}}\left[\frac{\sqrt{(1+v^{\alpha})}-1}{\sqrt{(1+v^{\alpha})}}\right] \tag{10}$$

VII. APPLICATIONS

In this section, some applications are given in order to explain the advantage of Sadik transform of error function for evaluating the improper integral, which contain error function.

7.1 Evaluate the improper integral $\boldsymbol{I = \int_0^\infty e^{-t}\, erf(\sqrt{t})\, dt}$.

We have $S\{erf(\sqrt{t})\} = \frac{1}{v^{\beta}}\int_0^\infty erf(\sqrt{t})\, e^{-tv^{\alpha}}\, dt$

$$\Rightarrow S\{erf(\sqrt{t})\} = \frac{1}{v^{\alpha+\beta}\sqrt{(1+v^{\alpha})}} \tag{11}$$

Taking $v^{\alpha} \to 1$ in above equation, we have

$$\frac{1}{v^{\beta}}\int_0^\infty e^{-t}\, erf(\sqrt{t})\, dt = \frac{1}{v^{\beta}.\sqrt{2}}$$

$$\Rightarrow I = \int_0^\infty e^{-t}\, erf(\sqrt{t})\, dt = \frac{1}{\sqrt{2}}$$

7.2 Evaluate the improper integral $\boldsymbol{I = \int_0^\infty e^{-(v-2)t}\, erf(\sqrt{t})\, dt}$.

We have $S\{erf(\sqrt{t})\} = \frac{1}{v^{\alpha+\beta}\sqrt{(1+v^{\alpha})}}$

Now by shifting theorem of Sadik transform, we have

$$S\{e^{2t}\, erf(\sqrt{t})\} = \left[\frac{1}{v^{\beta}(v^{\alpha}-2)\sqrt{(1+v^{\alpha}-2)}}\right]$$

$$\Rightarrow S\{e^{2t}\, erf(\sqrt{t})\} = \left[\frac{1}{v^{\beta}(v^{\alpha}-2)\sqrt{(v^{\alpha}-1)}}\right] \tag{12}$$

By the definition of Sadik transform, we have

$$S\{e^{2t}\, erf(\sqrt{t})\} = \frac{1}{v^{\beta}}\int_0^\infty e^{2t}\, erf(\sqrt{t})\, e^{-tv^{\alpha}}\, dt$$

$$\Rightarrow S\{e^{2t}\, erf(\sqrt{t})\} = \frac{1}{v^{\beta}}\int_0^\infty e^{-(v^{\alpha}-2)t}\, erf(\sqrt{t})\, dt \tag{13}$$

Now by equations (12) and (13), we get

$$\frac{1}{v^{\beta}}\int_0^\infty e^{-(v^{\alpha}-2)t}\, erf(\sqrt{t})\, dt = \frac{1}{v^{\beta}(v^{\alpha}-2)\sqrt{(v^{\alpha}-1)}}$$

Taking $\alpha \to 1$ in above equation, we have

$$\Rightarrow I = \int_0^\infty e^{-(v-2)t}\, erf(\sqrt{t})\, dt = \frac{1}{(v-2)\sqrt{(v-1)}}.$$

7.3 Evaluate the improper integral $\boldsymbol{I = \int_0^\infty e^{-t}\left\{\int_0^t erf(\sqrt{x})dx\right\}dt}$.

We have $S\{erf(\sqrt{t})\} = \frac{1}{v^{\alpha+\beta}\sqrt{(1+v^{\alpha})}}$

Now by the property of Sadik transform of integral of a function, we have

$$S\left\{\int_0^t erf(\sqrt{x})dx\right\} = \frac{1}{v^\alpha}\left[\frac{1}{v^{\alpha+\beta}\sqrt{(1+v^\alpha)}}\right]$$

$$\Rightarrow S\left\{\int_0^t erf(\sqrt{x})dx\right\} = \frac{1}{v^{2\alpha+\beta}\sqrt{(1+v^\alpha)}} \qquad (14)$$

By the definition of Sadik transform, we have

$$S\left\{\int_0^t erf(\sqrt{x})dx\right\} = \frac{1}{v^\beta}\int_0^\infty e^{-tv^\alpha}\left\{\int_0^t erf(\sqrt{u})du\right\}dt \qquad (15)$$

Now by equations (14) and (15), we get

$$\frac{1}{v^\beta}\int_0^\infty e^{-tv^\alpha}\left\{\int_0^t erf(\sqrt{u})du\right\}dt = \frac{1}{v^{2\alpha+\beta}\sqrt{(1+v^\alpha)}}$$

Taking $v^\alpha \rightarrow 1$ in above equation, we have

$$I = \int_0^\infty e^{-t}\left\{\int_0^t erf(\sqrt{x})dx\right\}dt = \frac{1}{\sqrt{2}}.$$

7.4 Evaluate the improper integral $I = \int_0^\infty e^{-2t}\left[\frac{d}{dt}erf(2\sqrt{t})\right]dt$.

We have $S\{erf(\sqrt{t})\} = \frac{1}{v^{\alpha+\beta}\sqrt{(1+v^\alpha)}}$

Now by change of scale property of Sadik transform, we have

$$S\{erf(2\sqrt{t})\} = \frac{1}{4}\left[\frac{1}{v^\beta\left(\frac{v^\alpha}{4}\right)\sqrt{\left(1+\frac{v^\alpha}{4}\right)}}\right]$$

$$\Rightarrow S\{erf(2\sqrt{t})\} = \frac{2}{v^{\alpha+\beta}\sqrt{(4+v^\alpha)}}$$

Now using the property of Sadik transform of derivative of a function, we have

$$S\left\{\frac{d}{dt}erf(2\sqrt{t})\right\} = v^\alpha\left[\frac{2}{v^{\alpha+\beta}\sqrt{(4+v^\alpha)}}\right] - 0$$

$$\Rightarrow S\left\{\frac{d}{dt}erf(2\sqrt{t})\right\} = \frac{2}{v^\beta\sqrt{(4+v^\alpha)}} \qquad (16)$$

By the definition of Sadik transform, we have

$$S\left\{\frac{d}{dt}erf(2\sqrt{t})\right\} = \frac{1}{v^\beta}\int_0^\infty e^{-tv^\alpha}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt \qquad (17)$$

Now by equations (16) and (17), we get

$$\frac{1}{v^\beta}\int_0^\infty e^{-tv^\alpha}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \frac{2}{v^\beta\sqrt{(4+v^\alpha)}}$$

Taking $v^\alpha \rightarrow 2$ in above equation, we have

$$\int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \frac{2}{\sqrt{6}}$$

$$\Rightarrow I = \int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \frac{2}{\sqrt{6}}$$

$$\Rightarrow I = \int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \sqrt{\frac{2}{3}}.$$

7.5 Evaluate the improper integral $I = \int_0^\infty e^{-5t}\left[erf\sqrt{t} * erf\sqrt{t}\right]dt$.

By convolution theorem of Sadik transform, we have

$$S\{erf\sqrt{t} * erf\sqrt{t}\} = v^\beta S\{erf(\sqrt{t})\}S\{erf(\sqrt{t})\}$$

$$= v^\beta\left[\frac{1}{v^{\alpha+\beta}\sqrt{(1+v^\alpha)}}\right]\left[\frac{1}{v^{\alpha+\beta}\sqrt{(1+v^\alpha)}}\right] = \frac{1}{v^\beta v^{2\alpha}(1+v^\alpha)} \qquad (18)$$

Now by the definition of Sadik transform, we have

$$S\{erf\sqrt{t} * erf\sqrt{t}\} = \frac{1}{v^{\beta}}\int_0^{\infty} e^{-tv^{\alpha}}\{erf\sqrt{t} * erf\sqrt{t}\}dt \quad (19)$$

Now by equations (18) and (19), we get

$$\frac{1}{v^{\beta}}\int_0^{\infty} e^{-tv^{\alpha}}\{erf\sqrt{t} * erf\sqrt{t}\}dt = \frac{1}{v^{\beta}v^{2\alpha}(1+v^{\alpha})} \quad (20)$$

Taking $v^{\alpha} \to 5$ in above equation, we have

$$\int_0^{\infty} e^{-5t}\{erf\sqrt{t} * erf\sqrt{t}\}dt = \frac{1}{150}$$

$$\Rightarrow I = \int_0^{\infty} e^{-5t}\{erf\sqrt{t} * erf\sqrt{t}\}dt = \frac{1}{150}.$$

VIII. CONCLUSIONS

In this article, we have successfully discussed the Sadik transform of error function. The given numerical applications in application section show the advantage of Sadik transform of error function for evaluating the improper integral, which contain error function. Results of numerical applications show Sadik transform give the exact solution without any tedious calculation work. In future, Sadik transform can be used in solving vibrating beam problems, heat and mass transfer problems.

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